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Functional Thinking in Mathematics Learning: What and How to Measure it?

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Abstrak

Berpikir fungsional menjadi prinsip dalam proses pembelajaran matematika di sekolah. Meskipun kebutuhan akan tinjauan berpikir fungsional semakin meningkat, namun hingga saat ini belum ada tinjauan literatur sistematis yang membahas tentang apa dan bagaimana mengukur berpikir fungsional dalam pembelajaran matematika. Tujuan dari penelitian ini adalah untuk menganalisis pentingnya berpikir fungsional dalam pembelajaran matematika dan bagaimana mengukur berpikir fungsional. Metode yang digunakan dalam penelitian tinjauan sistematik ini mengacu pada PRISMA. Total artikel yang terkumpul sebanyak 5.415.874. Berdasarkan tahapan PRISMA yang terdiri dari Identifikasi, Penyaringan, Kelayakan, dan Penyertaan, terdapat 63 artikel yang memenuhi syarat. Hasil review terhadap 63 artikel menunjukkan bahwa ada beberapa hal yang harus diperhatikan dalam mengembangkan berpikir fungsional dalam pembelajaran matematika seperti penggunaan masalah terbuka, penyelesaian masalah realistik, penggunaan teknologi, dan pemberian soal latihan rutin dan non rutin. Selanjutnya dalam mengukur berpikir fungsional dapat dilihat berdasarkan indikatornya yaitu menuliskan unsur selanjutnya berdasarkan pola sebelumnya (pola rekursif), menggunakan hubungan antar unsur untuk meneruskan hubungan ke unsur secara umum (hubungan kovarian), dan menyatakan hubungan antara dua unsur yang berbeda-beda dalam bentuk aturan fungsi. (korespondensi).

***Kata kunci*:** Functional Thinking, *Matematika, Pembelajaran*

Abstract

Functional thinking becomes a principle in the process of learning mathematics at school. Even though the need for a review of functional thinking is increasing, until now there has been no systematic literature review discussing what and how to measure functional thinking in mathematics learning. The aim of this study is to analyze the importance of functional thinking in mathematics learning and how to measure functional thinking. The method used in this systematic review study refers to PRISMA. The total articles collected were 5,415,874. Based on the PRISMA steps which consist of Identification, Screening, Eligibility, and Included, 63 articles met the requirements. The results of a review of 63 articles show that there are several things that must be considered in developing functional thinking in learning mathematics such as using open problems, solving realistic problems, using technology, and giving routine and non-routine practice questions. Furthermore, in measuring functional thinking, it can be seen based on the indicators, namely writing the next element based on the previous pattern (recursive pattern), using the relationship between elements to continue the relationship to elements in general (covariational relationship), and stating the relationship between two elements that vary in the form of function rules (correspondence).

Keywords: Functional Thinking, Mathematics, Learning

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1. INTRODUCTION

In mathematics, functions are used to model many real-world applications and functional thinking is seen as important for learning mathematics. Functional thinking is part of algebraic thinking and a fundamental component of algebraic thinking (Asghary et al., 2013; Lichti & Roth, 2018; Wilkie & Clarke, 2016). When an individual can choose to pay attention to two or more varying quantities and then start to focus on the relationship between these quantities, the individual is carrying out functional thinking activities (Pinto & Cañadas, 2017; Tanişli, 2011). Functional thinking is a principle in the process of learning mathematics in schools which refers to the modernization of subject matter and focuses on unifying various branches of school mathematics (Krüger, 2019; Lichti & Roth, 2019; Utomo

et al., 2021). In everyday life this dependency relationship can be found in the distance traveled by a car depending on speed and time.

Research on functional thinking is lagging behind in the new mathematics era and is mostly carried out by European countries. Other research that contributes to the development of functional thinking is research which states that there is a correspondence between cognitive and functional to explain students' understanding of mathematics, concepts, and problem-solving behavior (Cohors-Fresenborg et al., 2003; Stephens et al., 2017). Subsequent research is relatively small and limited to the relationship between different problem variables. Functional thinking must be nurtured earlier than elementary school and is increasingly learned in middle and high school mathematics. Some examples of fundamental functional relationships in elementary schools are students recognizing number patterns, while junior high school students focus more on geometric patterns, while for high school students students find differences in linear and quadratic functions (Gardiner, 2016; Lichti & Roth, 2019; Suryowati, 2021).

At the college level, the ability to think functionally becomes a basic ability in learning algebra and calculus. The results of this research are more focused on functional thinking in general, such as considering the relationship between variables in different problems. Even though functional thinking is one of the skills needed in the era of society 4.0 and towards 5.0 where functional thinking becomes the ability of students to execute functional programming in coding (Slavíčková & Vargová, 2018; Tuomi et al., 2018). This causes no review that focuses on what and how to measure functional thinking in learning mathematics. Therefore, in this systematic literature review, it aims to analyze the meaning and method of measuring functional thinking in mathematics learning based on empirical results. This literature review has never existed before, so it is hoped that this SLR can be an additional literature on functional thinking which is still limited.

2. METHODS

The method used in this systematic review study refers to the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA), namely identification, screening, eligibility, and inclusion (Moher et al., 2009). A literature search was conducted on June 24, 2023. A systematic literature search was conducted on the Scopus, ScienceDirect, Google Schoolar, and Teacher Reference Center document databases which have high-quality indexing standards and a good international reputation in the field of mathematics education. Relevant articles are obtained with various selected searches such as article titles, abstracts, and keywords. The search ring is show in Table 1.

Table 1. Search Ring

Databases	Item Search	Number of Articles
Scopus	*Functional AND Thinking*	8.989
	Functional AND Relationship	217
Science Direct	Functional Thinking	156.657
Google Schoolar	Functional Thinking in Education	5.250.000
Teacher Reference Center	Functional Thinking	11
Total		**5.415.874**

The search includes studies conducted at all levels of mathematics education and regardless of year of publication. The first step taken was to search 4 databases using Publish

and Perish to obtain 5,415,874 as shown in Table 1. Furthermore, the article database was reduced using the VOS Viewer based on similarities between titles from different websites, suitability of keywords with results, and suitability of articles with the scope of education to produce 216 articles, and in the end 63 articles were included in a systematic review.

3. RESULTS AND DISCUSSION

Results

In general, the articles that were published before, have not defined in detail the meaning of functional thinking, there are still many experts who use the opinions of previous experts in defining functional thinking. A total of 63 articles that meet the requirements were reviewed using the Vos Viewer to find definitions and ways to measure functional thinking. Articles about functional thinking mostly discuss elementary school students. However, in recent years, many experts have discussed functional thinking with the subject of high school students. Based on the review results, there are three keywords that appear most often, namely functional reasoning, generalizing the relationship between two variables, and making function rules. Functional thinking keywords is show in Table 2.

Table 2. Functional Thinking Keywords

Functional Thinking Keywords	Expert
Functional Reasoning	(Stephens et al., 2017); (M. L. Blanton & Kaput, 2011); (Pinto & Cañadas, 2018); (Cañadas et al., 2016);
Generalization of the Relationship between the Two Variables	(Pang et al., 2022); (Pinto & Cañadas, 2018); (Mceldoon & Rittle-Johnson, 2010); (Pang & Sunwoo, 2022); (Lichti & Roth, 2019); (Frey et al., 2022); (Krüger, 2019); (Utami et al., 2023), (Ayala-Altamirano & Molina, 2020); (Thompson & Carlson, 2017); (Rolfes et al., 2022)
Create Function Rules	(Pinto et al., 2022); (Utami et al., 2023)

Based on Table 2 show the results of the three most used keywords, it can be concluded that functional thinking is a function reasoning ability that generalizes the relationship between two variables to make function rules. In measuring functional thinking, many experts such as that functional thinking consists of three forms, namely recursive patterning (finding variations in the order of values), covariational thinking (analyzing two varying quantities simultaneously as an explicit and dynamic part of the function description), correspondence relationship (identifying the correlation between variables). However, from the existing articles there are no articles that discuss the aspects needed to develop functional thinking in learning mathematics and how to measure this functional thinking. Opinions from functional thinking must be cultivated earlier than elementary school and are increasingly studied in middle and high school mathematics. The development of research on functional thinking is also not too much.

Research on functional thinking was lagging behind in the new mathematics era and research on functional thinking at the beginning of the decade was mostly carried out by European countries, while the United States at that time was more interested in the ability to think mathematically. The results of the research from previous study concluded that functional thinking is needed in STEM because functional thinking is characterized as a special way of thinking in relationships, dependencies, and changes that are closely related to STEM, especially in everyday situations where two quantities are connected (Sproesser et al., 2022). Meanwhile, in 2023, functional thinking research focuses on expanding students' prior

knowledge to study general structures and relationships in functions and generalizing, representing, and justifying generalizations through the use of more abstract and symbolic expressions such as formulas (Chimoni et al., 2023). There are four categories in functions, namely (1) pattern tasks that require one to look for numerical patterns and sequences, (2) arithmetic operation set tasks, namely mathematical calculations of functions, (3) one-to-one corresponding tasks, namely comparing between elements and (4) pre-functional tasks, namely real-life problems that include two different quantities, for example, using letters to represent different quantities, and table problems that include proportional or linear relationships (Ding et al., 2022; Doorman et al., 2012).

According to other study there are levels in obtaining functional thinking, namely (1) functions as input-output assignments, namely the operational and computational characteristics of the function concept; (2) function as a dynamic process of covariation that relates the two changing quantities (Krišáková & Slabý, 2022; Thompson & Carlson, 2017); (3) function as a correspondence relationship by identifying the correlation between variables, using function rules; and (4) functions as mathematical objects by representing functions in various ways such as tables, graphs, or formulas. The way to Measure Functional thinking Based on Categories is show in Table 3.

Table 3. How to Measure Functional thinking Based on Categories

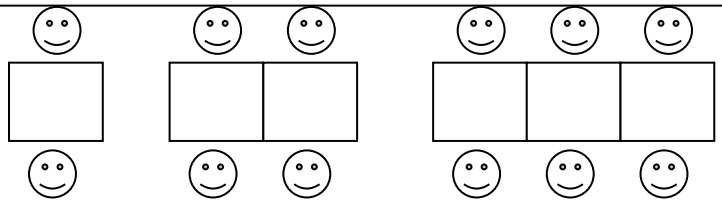
Category	Details
Recursive Pattern	Write down the next pattern by counting the elements
Covariational Relationship	Use the differences found to continue the sequence in finding relationships between elements and use relationships between elements to continue these relationships to elements in general
Correspondence	Expresses a relationship between two elements that vary in general based on properties such as numerical order.

In functional relationships, students can reflect on output variations that correspond to input variations. As for functional thinking, there are several categories such as interpretation of variables and the relationship between variables using covariation and correspondence approaches for generalization; and connections between representations to interpret relationships (Donevska-Todorova et al., 2022; Oliveira et al., 2021). Furthermore, in measuring functional thinking, three indicators can be built, namely recursive pattern, covariational relationship, and correspondence (M. Blanton et al., 2015; Pang et al., 2022; Stephens et al., 2017). Recursive patterning task is a way of thinking to find repeating units in a given pattern. Previous study suggests that in generalizing patterns, the initial activity is to distinguish the invariants of the various elements of the pattern . Patterns can occur within one object, within an ordered set of objects, or between two ordered sets of objects in a repeating pattern formula. Recursive patterning tasks are classified into geometric patterns (patterns using visual images) and numerical patterns (patterns using numbers) as shown in Table 4.

Table 4. Example of Recursive Patterning

Forms of Recursive Patterning	Example
Geometric patterns	
Numeric pattern	1,3,5,7,…,…,… 2,4,6,…,…,…

Covariation tasks understand the context of the problem and pay attention to two different quantities from the context to recognize the relationship between the two quantities. The initial step in covariance thinking is to pay attention to and understand each element as a varying quantity, see the pattern, identify pairs of values from the paired quantities, and coordinate variations.Example of covariant task is show in Figure 1.



What is the relationship between the number of tables and the number of people that can be seated? How many tables if there are 10 people?

Figure 1. Example of Covariant Tasks

The last indicator is the correspondence tasks which are the general relationship between two variables, which are specifically stated as function rules. Correspondence reasoning is essential to the function of teaching and learning in schools and colleges. Correspondence determines the output value associated with the input value simultaneously by identifying the relationship between elements in the form of function rules. Therefore, it is better to present non-consecutive values using a function table. When using function tables, students easily focus on changes and prevent students from using recursive thinking or covariate thinking. Example of correspondent task is show in Figure 2.

The following table is presented which describes the output value of the input entered into a function. Determine the function rules that describe the input and output relations!

Input	Output
3	7
1	3
4	9

Figure 2. Example of Corresponded Task

Discussion

An innovative learning environment can be created by forming functional thinking. The reasoning process used to construct and generalize patterns and relationships can be called functional thinking (Antari et al., 2013; Niswara et al., 2019). Functional thinking in general can be interpreted as a process of building, describing, and reasoning with and about functions. This type of thinking that focuses on the invariant relationship between two varying quantities/variables can also be called functional thinking. Functional thinking is a component of algebraic reasoning and part of students' mathematical reasoning (Martins et al., 2023; Pittalis et al., 2020). Functional thinking abilities require students' proficiency in understanding functions as objects and functions showing the same value regardless of how the function is represented. Based on the opinions of some of these experts, it can be concluded that functional thinking is an individual's ability to relate two quantities that are generalized into a form of function.

According to previous study when giving commands to smartphones and tablets, problems are clearly defined and separated into parts called functions (Koyuncu & Koyuncu, 2019). Then a combination of functions and each function that contains a command is used to

solve the problem. The concept of mathematical functions and functional thinking is an important idea in mathematics education (Lindenbauer, 2020). The concept of function can be used when teaching geometry which is an essential element of mathematics. Before implementing algorithms in coding or functional programming, students should have functional thinking abilities (Frith et al., 2020; Günster & Weigand, 2020).

Founder of functional thinking created guiding principles in teaching mathematics to unify various areas of school mathematics. In an algebraic context, a function is a mathematical statement that describes how two covary quantities consist of a domain, a codomain, and a rule in which each element in the domain is assigned to a single element of the codomain (Ayala-Altamirano & Molina, 2021; Weigand & Günster, 2022). Functional thinking is closely related to the concept of mathematical functions which can be accessed through mathematical representations because one's thoughts cannot be directly observed, which can only be inferred and interpreted from actions or verbalizations carried out to be brought back to a mathematical level. Functional thinking also discusses patterns or structures, generalizing and expressing generalizations, as well as understanding and using them (Niswara et al., 2019; Suartini et al., 2014).

One of the uses of functional thinking in learning mathematics is to create a mathematical model based on derived relationships. According to previous study activities in functional thinking can be done by analyzing, interpreting, and comparing various function representations, characterizing functions based on their properties (Ursini & Trigueros, 2001). There are several things that must be considered in developing functional thinking such as using technology, giving routine and non-routine practice questions, using image sequences to relate image elements and their positions, and solving realistic problems with the help of functions (Slavíčková & Vargová, 2018; Wei et al., 2022).

Functional thinking is a principle in the process of learning mathematics in schools which refers to the modernization of subject matter and focuses on unifying various branches of school mathematics (Oktafiani et al., 2020; Wedayanti & Wiarta, 2022). According to previous study selected open problems can be used by the teacher as a tool for a deeper understanding of the concept of function and for the development of functional thinking (Bulková Kristína & Čeretková Soňa, 2019). Activities in functional thinking can be carried out by analyzing, interpreting, and comparing various function representations, characterizing functions based on their properties, and solving realistic problems with the help of functions (Günster & Weigand, 2020; Krüger, 2019).

This research can provide insight into how to develop more effective teaching methods to facilitate functional thinking in mathematics learning. The results of this research can be used to improve mathematics curricula so that they better promote deep and functional understanding of concepts. This research has limitations that only apply to certain age groups, and cannot be applied generally to all levels of education. In addition, this research may be limited to certain samples, such as certain age groups or education levels, so the results may not be directly applicable to the wider population.

4. CONCLUSION

Based on the results of the literatur study review, the individual's ability to connect between two generalized quantities into a form of function is called functional thinking. In learning mathematics there are several things that must be considered in developing functional thinking such as using technology, giving routine and non-routine practice questions, creating mathematical models, and solving realistic problems. Activities in functional thinking can be done by analyzing, interpreting, and comparing various function representations, as well as characterizing functions based on their properties by means of

recursive patterns, covariational relationships, and correspondences. These three activities can be measured by the interpretation of variables and relationships between variables using generalizations and connections between representations to interpret relationships. It is hoped that the results of this literature review can become additional literature on functional thinking which is still limited.

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